

# Applications of QCD<sup>1</sup>

*Martin Beneke*  
*Theory Division, CERN*  
*CH-1211 Geneva 23, Switzerland*

## Introduction

Quantum chromodynamics (QCD), the theory of the strong interaction, attracts a large body of theoretical work and experimental investigation. “Applications of QCD” [theory] probably means that the former should have something to do with the latter. This is a severe restriction. It leaves out, for example, phenomena at high temperature and large baryon density, which have not been tested in terrestrial laboratories. Yet there have been fascinating developments in these directions, which reflect once more the complexity that follows from a Lagrangian as simple as  $-G^2/4 + \text{quarks}$ . In this talk I concentrate on high energy QCD processes, which means that at least some part of the process must be tractable perturbatively. Even within this narrow frame striving at completeness would do injustice to the diversity of the (sub-)field. The following gives a survey and assessment of recent theoretical results on selected topics. For details please consult original references and topical reviews. Apologies for omitting topics that should have been included, but have not been for various reasons (lack of time, competence, ...).

The conceptual basis for discussing QCD processes at large momentum transfer  $Q$  is provided by factorization:

$$d\sigma = d\hat{\sigma}(Q, \mu) \otimes F(\mu, \Lambda_{\text{QCD}}) + O(\Lambda_{\text{QCD}}/Q). \quad (1)$$

The first factor,  $d\hat{\sigma}(Q, \mu)$ , is insensitive to long distances of order  $1/\Lambda_{\text{QCD}}$ . It is computed in perturbation theory as scattering of quarks and gluons and depends only on

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the strong coupling  $\alpha_s$  and heavy quark masses. The second factor accounts for the fact that experiments are prepared and measurements are done far away ( $\gg 1\text{ fm}$ ) from the interaction point.  $F(\mu, \Lambda_{\text{QCD}})$  parametrizes this long-distance sensitivity in terms of process dependent quantities: vacuum condensates, parton distributions, fragmentation functions, light cone wave-functions and many more. In principle,  $F(\mu, \Lambda_{\text{QCD}})$  depends only on  $\alpha_s$  and light quark masses, but since  $\alpha_s(\Lambda_{\text{QCD}})$  is large, we cannot compute it in perturbation theory. However, being independent of the hard scattering process, the same  $F(\mu, \Lambda_{\text{QCD}})$  may appear in a generic class of processes. There are some fortunate cases in which long distance sensitivity appears only in  $O(\Lambda/Q)$  in (1). In these cases, we have particularly clean predictions, if  $Q$  is large enough. In general, we need to provide  $F(\mu, \Lambda_{\text{QCD}})$ . It can sometimes be computed non-perturbatively by numerical methods (“lattice QCD”). Or it may be approximated by models of low-energy QCD. More often, however, some measurements are used to determine  $F(\mu, \Lambda_{\text{QCD}})$ ; others are then predicted. This makes QCD seem to depend on many infrared parameters along with  $\alpha_s$ . It also implies iterations of theory and experiment to arrive at predictions.

Eq. (1) suggests a procedure: for any given large momentum transfer process (i) establish (1), identify  $F(\mu, \Lambda_{\text{QCD}})$ ; (ii) compute  $d\hat{\sigma}(Q, \mu)$  accurately; (iii) if  $F(\mu, \Lambda_{\text{QCD}})$  is known, predict  $d\sigma$ , otherwise determine  $F(\mu, \Lambda_{\text{QCD}})$ , if  $d\sigma$  is measured; (iv) check the accuracy of this procedure by addressing power corrections  $O(\Lambda_{\text{QCD}}/Q)$ . The outline of this talk is divided in sections according to this procedure rather than by topics, although in different order. Sect. 1 covers perturbative calculations, Sect. 2 power corrections. Perturbative expansions of  $d\hat{\sigma}(Q, \mu)$  often fail in special kinematic regions, but accurate results can be recovered upon all-order resummations. In Sect. 3 I discuss three representative examples of this situation. Finally, Sect. 4 is devoted to some processes for which factorization has been established more recently.

It is important to remind ourselves that working with QCD we take many things for granted which have never been proven: that (1), obtained from factorization properties of Feynman diagrams, holds non-perturbatively; that the operator product expansion holds non-perturbatively; that perturbative expansions are asymptotic; that lattice QCD approaches the correct continuum limit. The overall picture of consistency that has emerged in applications of QCD suggests a pragmatic attitude towards these problems. However, the questions remain.

## 1 Perturbative calculations

For long-established QCD processes there are no easy perturbative calculations any more. Increasing the accuracy by one order in  $\alpha_s$  has become technically demanding, usually requiring extensive or automated algebraic manipulations by computers and/or numerical computing. The complications increase by increasing the number

of loops, or the number of mass scales or external legs.

## 1.1 More loops

Totally inclusive quantities are related to imaginary parts of correlation functions. This avoids infrared divergences in intermediate expressions. Such quantities are candidates for fully automated evaluation [1].

The  $\alpha_s^3$  correction to  $e^+e^- \rightarrow \text{hadrons}$  (massless quarks and gluons) and related observables, and to some deep inelastic scattering sum rules, have been known for some time [2]. More recently, the QCD  $\beta$ -function [3] and quark mass anomalous dimension [4] have been computed at 4-loop order.

These results use that any 3-loop, massless, 2-point integral is calculable in dimensional regularization. The most important tools are the integration-by-parts method [5], infrared re-arrangement [6] which reduces the calculation of the 4-loop pole part to the above class of diagrams, and powerful computers that handle the algebra connected with about  $10^4$  Feynman diagrams. Another important class of diagrams which is generically calculable is 3-loop, massive, vacuum bubble diagrams [7]. There is no obvious way to extend these results to one more loop.

## 1.2 More scales

Observables that depend on more than one kinematic invariant or a kinematic invariant and quark masses are difficult, even if they are totally inclusive. A method that has led to a number of interesting new results is based on asymptotic expansions in a ratio of scales, such that each term in the expansion is a single-scale integral that is analytically solvable. This method can be used even if the expansion parameter is not small if many terms in the expansion can be obtained and if the radius of convergence is sufficiently large or convergence can be improved by Padé approximants.

Asymptotic expansions can be performed (i) for large external momenta, small masses or for large masses, small external momenta [8]; (ii) around mass shell [9]; (iii) near thresholds [10] or in  $t/s$  for  $2 \rightarrow 2$  scattering; (iv) in Sudakov limits [11]. These expansions are done on the integrand level. The fact that loop momenta cover all scales implies that, in general, extra terms have to be added to the Taylor expansion of the integrand.

A nice example to illustrate the method is the 3-loop coefficient in the relation between the pole mass and the  $\overline{\text{MS}}$  mass of a heavy quark [12]. This requires 3-loop on-shell integrals, which are not known. Instead expand the quark self-energy around external momentum  $p^2 = 0$ , which reduces the problem to 3-loop vacuum bubbles, which are calculable. Then put  $p^2/m^2 = 1$  and use Padé approximants. Expansion to order  $(p^2/m^2)^{14}$  (plus information from the opposite limit  $p^2 \gg m^2$ ) gives  $r_3 = 3.10 \pm 0.06$  for the coefficient at order  $\alpha_s^3$  ( $n_f = 4$ ). (In retrospect, this turns

out to be a “bad” example, because the 3-loop on-shell integrals are in fact exactly calculable [13, 14]. The exact number is  $r_3 = 3.0451 \dots$  [14], in nice agreement with the previous semi-analytic result.)

Applications of this method up to now concern quantities with internal masses or on-shell lines.  $e^+e^- \rightarrow b\bar{b}X$  has been obtained at order  $\alpha_s^2$  for general  $q^2$  [15] and in an expansion near threshold [16]. The  $\alpha_s^2$  corrections to inclusive heavy quark decays has been calculated for  $b \rightarrow cl\nu$  [17],  $t \rightarrow bW$  [18] and  $b \rightarrow ul\nu$  [19]. The last result is particularly impressive, because the asymptotic expansion is obtained algebraically to all orders. It is then resummed to an exact result.

### 1.3 More legs

Higher order jet calculations pose a different sort of challenge, because the kinematics becomes complicated (as the number of jets increases), and because the calculation is done on the amplitude level. Infrared singularities cancel in an intricate way, or are factorized into parton densities (fragmentation functions) after cancellations. Almost certainly the final result is obtained after numerical integration.

Relatively recent results include NLO corrections to  $e^+e^- \rightarrow 4$  jets [20],  $e^+e^- \rightarrow 3$  jets with quark mass effects [21], which provide us with a first, yet imprecise, evidence of scale-dependence of the bottom quark mass [22]. Partial NLO results exist on  $p\bar{p} \rightarrow 3$  jets [23]. The full result is supposed to be completed soon.

### 1.4 Towards NNLO jets

The conceptual and technical frontier is set by NNLO jet calculations, the basic process being  $2 \rightarrow 2$  ( $pp \rightarrow 2$  jets or 1 jet inclusive,  $pp \rightarrow \gamma\gamma X$ ) or  $1 \rightarrow 3$  ( $e^+e^- \rightarrow 3$  jets). NNLO calculations provide detailed insight into jet structure and a better determination of  $\alpha_s$ . In  $e^+e^- \rightarrow 3$  jets they are important to understand the interplay between perturbative and power corrections.

There are several components to the NNLO jet project. The amplitudes have to be computed, which include 2-loop 4-point diagrams. Amplitudes with five and six partons have to be integrated analytically over the singular regions of phase space. After cancellation of infrared divergences, the remaining phase space integrals have to be evaluated numerically efficiently. There has been progress on many of these components recently.

Because of the integration over singular regions of phase space even tree amplitudes are non-trivial. In  $2 \rightarrow 4$  tree amplitudes one encounters a new situation, when two partons become simultaneously soft or three partons become collinear. The last case (squared and integrated over phase space) gives rise to a new class of splitting amplitudes (functions), when one parton decays into three collinear partons, which generalize the usual splitting functions. All soft, collinear, and mixed, limits have now

been analyzed [24]. Likewise, although the 1-loop five-point amplitudes are known in four dimensions, this is insufficient, because the two jet cross sections includes configurations where two of the partons are not resolved. Making use of the universality of soft and collinear limits, these amplitudes are now known to all orders in the dimensional regularization parameter  $\epsilon$  in those kinematic regions, where the phase space integration is singular [25].

The most difficult amplitude is the 2-loop virtual correction to the basic  $2 \rightarrow 2$  (or  $1 \rightarrow 3$ ) process. Until very recently, it has been unclear whether the basic scalar double box integrals are analytically calculable. In a stunning calculation [26] an analytic result was obtained for the planar double box integral, expressed in terms of elementary special functions, and an algorithm was provided to compute the integral with arbitrary numerator [27]. It is equally surprising that this result was obtained by elementary methods: the  $\alpha$ -representation and Mellin-Barnes transformation and summations of multiple sums obtained after taking the Mellin-Barnes integrations. The crossed double box was subsequently calculated [28] using the same methods. The numerator algebra, connected to multiple products of three-gluon vertices, is, however, highly non-trivial, and remains to be done. Methods, based on helicity amplitudes, colour decomposition, special gauges and unitarity exist to simplify the task [29]. Up to now this has been completed in a toy  $N = 4$  supersymmetric theory (leaving the scalar integral unevaluated) [30], and more recently for the maximally helicity violating amplitude in QCD [31].

Many of these results can be used also for NNLO corrections to  $e^+e^- \rightarrow 3$  jets. However, the 2-loop double box integrals with one off-shell external leg are not yet known. The infrared singularities at order  $1/\epsilon^{4,3,2}$  are known [32], but the structure of  $1/\epsilon$  poles remains to be elucidated.

In my opinion, the results that have been achieved over the past two years make success predictable, at least for NNLO 2 jets. On the other hand, many hard algebraic and numerical tasks remain to be done. Even with a concerted effort the relevant time scale is years rather than months. However, this is clearly a beautiful case, where most of the most advanced techniques for perturbative QCD calculations merge into a single project.

## 1.5 NNLO parton evolution

NNLO jets require NNLO parton distributions. Evolution of these parton distributions requires the NNLO DGLAP splitting functions. The complete NNLO splitting functions are still unknown, although some moments have been computed [33] some time ago and further constraints exist in the large- $x$  and small- $x$  limit.

Large evolution means large  $Q^2$ , since parton distributions are typically determined experimentally at some low scale. Large  $Q^2$  means large  $x$  and hence the known moments may already provide accurate information. Indeed, first construc-

tions of approximate NNLO non-singlet splitting functions have appeared that make use of the known information [34]. The constraints at small  $x$  turn out to be quite weak, but there seems to be little uncertainty for  $x > 0.1$ . When the splitting function is folded with a typical parton distribution this range increases to  $x > 0.02$ .

## 2 Beyond leading power

Perturbative expansions, if computed to arbitrarily high order, ultimately diverge. They become useless beyond a certain order, unless they are summed. In recent years, we have learnt to turn this embarrassment into a benefit, since the pattern of divergence tells us something about the scaling of power corrections  $(\Lambda_{\text{QCD}}/Q)^n$  to a hard scattering cross section. A particularly interesting type of divergence, called infrared renormalon, is related to integration over small loop momentum in Feynman integrals [35]. Roughly speaking, there is a relation between perturbative long distance sensitivity, the size of perturbative coefficients in higher orders, and the scaling of non-perturbative power corrections [36]. For inclusive deep inelastic scattering,  $n = 2$ , and one recovers higher twist corrections predicted by the operator product expansion.

### 2.1 Event shape observables and energy flow

For other, less inclusive, observables, such as event shape variables in  $e^+e^-$  and  $ep$  collisions, one often finds  $n = 1$  [37, 38, 39]. Since these variables are order  $\alpha_s$  perturbatively, they are prone to large non-perturbative (and perturbative) corrections. They have been investigated intensively over the past two years, theoretically and experimentally.

The leading power correction originates from soft partons emitted from a fast, nearly back-to-back  $q\bar{q}$  pair. Write

$$\langle S \rangle = \langle S^{\text{pert.}}(\mu_I) \rangle + \frac{\mu_I}{Q} \langle S^{NP}(\mu_I) \rangle + O(\Lambda_{\text{QCD}}^2/Q^2) \quad (2)$$

for an average event shape variable  $S$ . The experimentally measured energy dependence of  $\langle S \rangle$  clearly supports the existence of a  $1/Q$  power correction with a reasonably sized normalization  $\langle S^{NP}(\mu_I) \rangle$ , which is non-perturbative. An interesting hypothesis (also applied to event shape distributions [40]) states that the non-perturbative corrections are universal, i.e.  $\langle S^{NP}(\mu_I) \rangle \propto c_S \bar{\alpha}(\mu_I)$ , where  $c_S$  is observable dependent, but calculable and  $\bar{\alpha}(\mu_I)$  is non-perturbative but independent of  $S$  [38]. (This applies to thrust, jet masses and the  $C$  parameter. Other event shapes, such as jet broadenings, involve complications [41].) Four parton final states with two soft partons have also been investigated [42, 43, 44]. Remarkably, one finds that  $c_S$  is rescaled by the same factor for a variety of shape observables [42, 44]. The universality hypothesis

has led to a number of instructive experimental tests. Recent results on average event shapes and event shape distributions in  $e^+e^-$  annihilation [45] and DIS [46] tend to confirm the hypothesis within the expected accuracy. However, the fact that the value of  $\alpha_s$ , fitted simultaneously to each  $S$ , is somewhat unstable indicates that the present understanding is not perfect.

Universality may hold for a special class of observables, but it would be surprising, if it held in general. What is needed to shed light on the issue is a factorization theorem for soft gluons beyond leading power. Recall that factorization theorems for event shapes usually demonstrate that soft gluon corrections cancel at leading power. We are now interested in the leading contribution that is left over after this cancellation.

In [47] the problem is approached in terms of energy flow of soft particles. The universal, non-perturbative objects relevant to the two-jet limit ( $q\bar{q}$  plus soft partons) are

$$G(\vec{n}_1, \dots, \vec{n}_k; \mu_I) = \langle 0 | W^\dagger \prod_{i=1}^k \mathcal{E}(\vec{n}_i) W | 0 \rangle, \quad (3)$$

where  $\mathcal{E}(\vec{n}_i)$  measures soft energy flow in the direction of  $\vec{n}_i$ ,  $\mu_I$  is a factorization scale that defines what “soft” means, and  $W$  denotes a product of eikonal lines for the energetic  $q\bar{q}$  pair. The  $G(\vec{n}_1, \dots, \vec{n}_k; \mu_I)$  are horribly complicated objects and it is hardly conceivable that they could ever be extracted from measurements. However, the fact that they are independent of the hard scale  $Q$  already entails interesting predictions. For example, event shape distributions can be expressed as a convolution of a perturbative distribution and a non-perturbative  $Q$ -independent, but observable-dependent “shape function”, that follows from these energy flow correlation functions. Event shape averages can be represented as

$$\langle S \rangle_{1/Q} = \int d\vec{n} w_S(\vec{n}) G(\vec{n}), \quad (4)$$

with a calculable weight function  $w_S(\vec{n})$ . The single energy flow correlation function  $G(\vec{n})$  can in principle be determined from the leading power correction to the energy-energy correlation.

I find this a promising step towards understanding soft power corrections. The concept of energy flow is clearly important and deserves more attention, as it corresponds directly to calorimetric measurements. Observables that can be represented in terms of energy flow are automatically infrared safe. They may also be defined non-perturbatively and therefore be amenable to a more systematic analysis of power corrections [48].

## 2.2 OPE of the plaquette

There are also things that don’t work as expected. Consider the operator product expansion (OPE) of the plaquette expectation value in pure gauge theory at finite

lattice spacing, i.e. the inverse lattice spacing takes the role of the scale  $Q \gg \Lambda_{\text{QCD}}$ . The OPE gives

$$\langle \text{plaquette} \rangle = \sum_{n=1} c_n \alpha_s^{\text{latt}}(Q)^n + \frac{C(\alpha_s^{\text{latt}})}{Q^4} \langle \frac{\alpha_s}{\pi} GG \rangle + \dots \quad (5)$$

The coefficients  $c_n$  of the perturbative expansion have been computed to 8th order numerically [49]. After transformation to a continuum coupling definition, the coefficients exhibit the expected infrared renormalon growth. Summing the series approximately should give an accuracy of order  $1/Q^4$ , hence subtracting the summed series from non-perturbative Monte Carlo data for  $\langle \text{plaquette} \rangle$ , the remainder should scale as  $1/Q^4$ , consistent with the scaling of the gluon condensate term.

Contrary to this expectation, the remainder is found to approach a perfect  $1/Q^2$  scaling behaviour [50]. Since the OPE is one of the few tools we have to go beyond perturbation theory, this is clearly something we should understand. There may be subtleties with the transformation to the continuum scheme, since this transformation is not known to 8th order or may also have power corrections. The effective action at finite lattice spacing contains an infinite set of higher dimension operators. Could these add up to a  $1/Q^2$  power correction [51] so that the result is a lattice artefact? But there may be less profane explanations such as power corrections from short distances that affect coefficient functions (and therefore would not contradict the OPE) [52]. This possibility is not ruled out by any argument. It presents a fundamental question that challenges our understanding of non-perturbative short-distance expansions. It would also have implications for the phenomenology of power corrections to current correlation functions. For these reasons, the problem raised by [50] should be cleared up!

### 3 Perturbative resummations

Returning to perturbative expansions, it is not unusual that a perturbative expansion in  $\alpha_s$  breaks down, even though the coupling constant is small. This happens because the smallness of the coupling constant is compensated by a large kinematic invariant. In effect, one is dealing with a multi-scale problem. If all scales are large compared to  $\Lambda_{\text{QCD}}$ , the problem is perturbative and may be subjected to systematic all-order resummations. The kinematic conditions leading to the breakdown of perturbation theory can be quite different and the resummations reflect completely different physics. In this section I discuss three examples of such resummations, where progress has been made over the past two years.



### 3.1 Parton thresholds

A familiar source of large kinematic corrections is related to partonic thresholds. Consider the differential cross section

$$d\sigma = \sum_{i,j} f_{i/A} \otimes f_{j/B} \otimes d\hat{\sigma}_{ij \rightarrow f} \quad (6)$$

for a hard hadron-hadron collision. Large logarithms appear in  $d\hat{\sigma}$ , when the cms energy  $\hat{s}$  of  $i + j$  is just large enough to produce a given final state. For example, in production of a massive vector boson with mass  $Q$ , the leading correction is  $\alpha_s^n (\ln^{2n-1}(1-z))/[1-z]_+$  at order  $\alpha_s^n$ , where  $z = Q^2/\hat{s}$ , and perturbation theory breaks down for  $z \rightarrow 1$ .

In this case large logarithms originate from the lack of phase space for real emission and the incomplete cancellation of sensitivity to collinear and soft momentum. Because of this relation the structure of these logarithms is well understood. The logarithms exponentiate and can be resummed:

$$\int dz z^{N-1} d\hat{\sigma}(z) = H(\alpha_s) \exp [\ln N g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \dots] + O(1/N). \quad (7)$$

This resummation was worked out at next-to-leading logarithmic order (i.e. including  $g_2(\alpha_s \ln N)$ ) some years ago for  $2 \rightarrow 1$  processes (massive vector boson production) [53] and  $1 \rightarrow 2$  processes (event shape variables in  $e^+e^-$  in the 2-jet limit) [54].

Next-to-leading logarithmic resummation has now been extended to  $2 \rightarrow 2$  scattering processes [55, 56]. Several new complications appear in this case. Since the underlying hard process depends on an additional kinematic invariant,  $(-\hat{t})/\hat{s}$ , so do the functions that appear in the exponent of (7). Furthermore, the  $2 \rightarrow 2$  amplitude contains several colour amplitudes and since soft gluon emission carries away colour, these amplitudes mix, turning the exponential into a matrix exponential on the independent colour amplitudes. While the structure of resummation remains thus the same, the technical complications make the formalism more difficult to apply in practice.

Fortunately, simplifications occur for total cross sections. NLL resummed results have been presented for heavy quark production [56, 57] and prompt photon production [58]. For di-jet production at large transverse momentum, the formalism is in principle complete, but it has not yet been implemented [59]. It turns out that at energies of interest for heavy quark production and prompt photons, the effect of resummation is typically small, i.e. within the renormalization scale variation of a fixed order NLO calculation. The real benefit of resummation is a significant reduction of this scale dependence compared to NLO QCD, and hence, probably, the theoretical uncertainty. The  $E_T$  spectrum of prompt photons at low  $E_T$  remains in disagreement

with the data [60]. Since at  $E_T \approx$  several GeV power corrections in  $1/E_T$ , or intrinsic transverse momentum, can be very important, this is hardly a serious issue. It is, however, a serious problem for determining the gluon distribution at large  $x$ .

### 3.2 Non-relativistic

A different kind of partonic threshold is encountered in heavy quark production in  $e^+e^-$  annihilation. When the cms energy is just larger than  $4m_Q^2$ , the quark and antiquark move at small relative velocity and attract each other through a strong Coulomb force, even if  $\alpha_s$  is small. Formulated as a perturbative resummation problem, we need the terms

$$R_{e^+e^- \rightarrow Q\bar{Q}X} \sim v \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{v} \right)^n \cdot \{1(\text{LO}); \alpha_s, v(\text{NLO}); \alpha_s^2, \alpha_s v, v^2(\text{NNLO}); \dots\} \quad (8)$$

at leading order (LO), next-to-leading order (NLO), etc., where  $v$  is the small relative velocity.

The LO resummation is done by solving for the Green function of the Schrödinger equation with the Coulomb potential. To be more systematic, such concepts from quantum mechanics have to be derived from QCD, incorporating correctly the short-distance structure of QCD. This is done by a sequence of non-relativistic effective field theories. Quarks and gluons can be classified as hard (h), soft (s), potential (p) and ultrasoft (us) [10]. Then these modes are integrated out successively, according to the scheme  $\mathcal{L}_{\text{QCD}}[Q(h, s, p); g(h, s, p, us)] \rightarrow \mathcal{L}_{\text{NRQCD}}[Q(s, p); g(s, p, us)] \rightarrow \mathcal{L}_{\text{PNRQCD}}[Q(p); g(us)]$ , passing from QCD to non-relativistic QCD [61] to potential-non-relativistic QCD [62]. The equation of motion of PNRQCD is exactly the Schrödinger equation, with corrections to it that encode the information of the short-distance modes that have been integrated out.

With the help of this method the NNLO resummation has been performed. This leads to first principle NNLO calculations of  $t\bar{t}$  production near threshold (in  $e^+e^-$  collisions) [63, 64, 65, 66]. The NNLO correction turns out to be very important and has led to the conclusion that it is the  $\overline{\text{MS}}$  top quark mass rather than the pole mass that can be determined more accurately, though indirectly, from the cross section near threshold [64, 65]. Another important application concerns the determination of the  $b$  quark mass from  $e^+e^- \rightarrow b\bar{b}X$  [67, 68]. The recent analyses [68] that take care of adequate bottom mass renormalization prescriptions converge towards a common value for the bottom quark  $\overline{\text{MS}}$  mass, which I average as  $\overline{m}_b(\overline{m}_b) = 4.23 \pm 0.08 \text{ GeV}$ . The centre of attention is now on understanding logarithmic corrections in  $v$  [64, 69].

### 3.3 High energy, small $x$

The high energy limit  $s \gg Q^2$  of QCD cross sections is an old, yet unsolved problem. Large logarithms can appear either in the high-energy limit of hard partonic reactions, such as in  $\gamma^*\gamma^*$  scattering or forward jet production, or in the small- $x$  behaviour of parton distributions and their evolution. The leading logarithms  $(\alpha_s \ln s/Q^2)^n$  have been summed long ago by means of the BFKL equation [70]. This leads to cross sections that rise as  $s^{\bar{\alpha}_s/4 \ln 2}$  ( $\bar{\alpha}_s = N_c \alpha_s/\pi$ ) with energy. For many years most theoretical work has been concerned with the physical mechanism that would make the high energy limit compatible with unitarity, but a quantitative theory has not yet emerged. Most of the recent activity in small- $x$  physics has however been inspired by the completion of the NLO correction to the BFKL kernel [71], and its interpretation. The following discussion concentrates on this aspect.

Recall that phenomenological applications of LO BFKL theory have remained ambiguous or unsuccessful. HERA data on the gluon density indicates that DGLAP evolution works well, in fact too well, down to  $x \sim 10^{-6}$ . No resummation of  $\ln x$  corrections to the evolution kernels is required. There is some flexibility in the input gluon distribution, nevertheless the message is that departures from DGLAP cannot be large. Virtual photon scattering has been measured at LEP [72]. Even allowing for the fact that LO BFKL may not predict the normalization of the cross section well, the observed energy dependence is less steep than predicted. Forward pion production at HERA [73] may be described by LO BFKL, but other interpretations of the data seem possible.

It is therefore clearly interesting to see how NLO corrections affect this comparison. In the high energy limit, the cross section factorizes schematically as

$$\sigma = \int \frac{d^2 k_1}{k_1^2} \Phi_A(k_1) \frac{d^2 k_2}{k_2^2} \Phi_B(k_2) \int \frac{d\omega}{2\pi i} \left( \frac{s}{k_1 k_2} \right)^\omega G_\omega(k_1, k_2), \quad (9)$$

where  $A$  usually represents a virtual photon and  $B$  a virtual photon or a proton. In the latter case the impact factor  $\Phi_p(k_2)$  is not perturbatively calculable.  $k_{1,2}$  denote transverse momenta of the scattering objects,  $k \sim Q$  for virtual photons,  $k \sim \Lambda_{\text{QCD}}$  for protons. The factorized form (9) is believed to hold to next-to-leading logarithmic order, but beyond this order there are terms that cannot be associated with the four-(reggeized)-gluon Green function  $G_\omega(k_1, k_2)$ .  $G_\omega(k_1, k_2)$  satisfies the BFKL equation [70]

$$\omega G_\omega(k_1, k_2) = \delta^{(2)}(k_1 - k_2) + \int \frac{d^2 k}{\pi} K_\omega(k_1, k) G_\omega(k, k_2). \quad (10)$$

Roughly speaking, the leading order kernel  $K_\omega(k_1, k)$  sums a single gluon ladder exchanged between  $A$  and  $B$  with emissions ordered in longitudinal momentum. The NLO correction has to account for all configurations in which one power of  $\ln x$  is lost.

Partial results have been collected over many years and the full NLO correction has finally been completed [71]. It is usually presented through the action of the kernel on a set of test functions:

$$\int d^2k' K_\omega(k, k') \left( \frac{k'^2}{k^2} \right)^{\gamma-1} = \bar{\alpha}_s \chi_0(\gamma) \left[ 1 - \beta_0 \bar{\alpha}_s \ln \frac{k^2}{\mu^2} \right] + \bar{\alpha}_s^2 \chi_1(\gamma). \quad (11)$$

In the saddle point approximation for the inverse Mellin integrals, treating  $\bar{\alpha}_s$  as small, the energy growth  $s^\lambda$  of hard high energy cross sections is then determined by

$$\lambda = \bar{\alpha}_s \chi_0(1/2) + \bar{\alpha}_s^2 \chi_1(1/2) = \bar{\alpha}_s 4 \ln 2 [1 - 6.5 \bar{\alpha}_s], \quad (12)$$

ignoring the scale dependent part of the kernel. The NLO correction is huge, large enough to modify qualitatively the conclusions drawn from leading order, which is good. At the same time, the NLO kernel taken at face value leads to non-sense results [74], unless  $\bar{\alpha}_s \leq 0.05$ , which is unrealistically small.

Much effort has gone into the question whether the NLO result invalidates the BFKL resummation programme as a whole. To answer this question one has to go beyond a systematic resummation of high energy logarithms. Such a step is unavoidably ambiguous and needs to be motivated by physics arguments. It appears that much of the NLO characteristic function  $\chi_1(\gamma)$ , even near  $\gamma = 1/2$ , can be understood from the singularities at  $\gamma = 0$  and  $1$ . Note from (11) that these singularities correspond to transverse logarithms. The leading singularities  $1/\gamma^{3,2}$ ,  $1/(1-\gamma)^{3,2}$  are related to the symmetric energy scale  $k_1 k_2$  chosen in (9), the running coupling and the non-singular terms of the LO DGLAP splitting function. Remarkably,  $\chi_1(\gamma)$  is extremely well reproduced just by keeping these singularities.

This suggests that these singularities should be summed to all orders. Unphysical transverse logarithms generated by the symmetric energy scale can be removed [75] by replacing

$$\chi_0(\gamma) \rightarrow \chi_0^\omega(\gamma) = 2\psi(1) - \psi(\gamma + \omega/2) - \psi(1 - \gamma + \omega/2). \quad (13)$$

Although not unique, this seems to be a particularly natural choice. After performing this replacement, the NLO correction is reduced, though not small. Further support for this resummation arises from the possibility to introduce a “rapidity veto”  $y_{i+1} - y_i > \Delta$  [76], which is essentially a hard cut-off on the momentum region, where the ordering in rapidity was not a good approximation in the first place. After resummation, the  $\Delta$ -dependence is small and the NLO correction moderate for all  $\Delta$  [77], which indicates that the resummed kernel is less sensitive to momentum regions where the approximations necessary to derive it are not valid.

The remaining  $\gamma$ -singularities are double poles. Two further modifications beyond NLO small- $x$  logarithms need to be performed to take care of them. First, rather than improving the DGLAP anomalous dimension by small- $x$  logarithms, we can take

the opposite point of view and improve  $\chi(\gamma)$  by taking into account all information on collinear logarithms [78, 79, 80]. In this way one can arrange, in addition, for momentum conservation, which requires vanishing anomalous dimension at  $\omega = 1$ . Second, the 1-loop evolution of  $\alpha_s$  can be taken into account exactly, rather than perturbatively as in (11). There are two cases to consider, symmetric processes with  $k_1 \sim k_2 \gg \Lambda_{\text{QCD}}$  [81] and asymmetric processes [82, 79]. In the latter case, with  $Q \sim k_1 \gg k_2 \sim \Lambda_{\text{QCD}}$  as for deep inelastic scattering, one must also apply collinear factorization to the four gluon Green function, such that [83]

$$G_\omega(k_1, k_2) = F_\omega^{\text{UV}}(k_1) \cdot F_\omega^{\text{IR}}(k_2) + O(k_2^2/k_1^2). \quad (14)$$

The dependence on the non-perturbative low momentum evolution of the running coupling is factorized into  $F_\omega^{\text{IR}}(k_2)$ , which can be absorbed into the input gluon distribution. This part remains beyond perturbative control, although it may well control the actual small- $x$  behaviour of the gluon distribution. On the other hand, only  $F_\omega^{\text{UV}}(k_1)$  is  $Q$ -dependent and hence determines the evolution of the gluon density.

There seems yet not to be an unanimous opinion on which of these aspects is most important. For example, [80] argues that  $\lambda$  should be considered as a non-perturbative parameter, while [79] takes a less agnostic attitude. Ref. [82], on the other hand, emphasizes the role of the running coupling, demonstrating that the effective scale of  $\alpha_s$  in the anomalous dimension increases as  $x$  decreases, because of ultraviolet diffusion. It is also claimed that this leads to an improved fit to structure function data compared to a standard DGLAP fit, which is definitely interesting. Despite these different viewpoints, theory clearly seems to be on the right track, as the results consistently point towards a smaller (but positive) hard pomeron intercept compared to LO BFKL. The resummed gluon anomalous dimension is also close to the DGLAP one down to rather small moments. It will be interesting to see consolidation of this field and the first true NLO+improved BFKL predictions for physical processes (which needs as yet unknown NLO impact factors).

## 4 Novel factorization “theorems”

In the past sections I discussed hard scattering processes which have been known as such. But for other processes factorization of its short-distance part has been established only recently. Often factorization comes at the expense of introducing new non-perturbative parameters. Even if these parameters are not accessible immediately, much is gained in terms of conceptual clarity. In this section I discuss three examples of such “new” applications of QCD.

## 4.1 Hard diffraction

A particularly nice example is hard diffraction [84]. Discovered in hadron-hadron collisions by UA8 about a decade ago [85], after the inspiring work of [86], the extent to which hard diffraction is a hard process, has remained rather unclear. This has changed completely with the arrival of accurate data on hard diffraction in  $ep$  scattering [87], the demise of Regge terminology, and the realization that hard diffraction in DIS can be described in close analogy with inclusive DIS [88, 89].

In hard diffractive DIS,  $\gamma^*p \rightarrow Xp$ , the proton scatters (quasi-)elastically off a virtual photon, which fragments into a colour neutral cluster  $X$ . The scattered proton is usually not detected, but since it typically loses only a small fraction of its momentum, the event is identified by a large gap in rapidity between  $p$  and  $X$ . About 10% of all DIS events are rapidity gap events. Furthermore, hard diffraction is not suppressed with  $1/Q^2$  relative to inclusive DIS.

In close analogy with inclusive DIS, the diffractive cross section factorizes into a short-distance cross section and a diffractive parton distribution [89, 90]:

$$\frac{d\sigma^D(x, Q^2, \xi, t)}{d\xi dt} = \sum_{i=q,g} \int_x^\xi dy \hat{\sigma}^{\gamma^*i}(Q, x, y; \mu) \frac{df_i^D(y, \xi, t; \mu)}{d\xi dt}. \quad (15)$$

The diffractive parton distribution  $f_i^D(y, \xi, t; \mu)$  represents the probability to find parton  $i$  in the proton with momentum fraction  $y$  under the condition that the proton stays intact and loses longitudinal momentum fraction  $\xi$ . Note that this definition makes no reference to Regge factorization or the pomeron. Neither does it make reference to a rapidity gap  $\Delta y$ , which follows from kinematics alone when  $\xi$  is small:  $\Delta y \sim \ln(1/\xi)$ . The hard scattering occurs on a single parton as in ordinary DIS. The dynamics that is responsible for the formation of a colour-singlet cluster is non-perturbative and therefore part of the definition of the diffractive parton distribution.

The physical picture of hard diffraction is perhaps clearest in the proton rest frame and reminiscent of the “aligned jet model” [91, 92]. In the proton rest frame, at small Bjorken  $x$ , the virtual photon splits into a  $q\bar{q}$  pair long before it hits the proton. The  $q\bar{q}$  wave-function of the virtual photon suppresses configurations in which one of the quarks carries almost all momentum. Yet it is these configurations that give rise to a large diffractive cross section, because the wave-function suppression is compensated by the large cross section for the scattering of a  $q\bar{q}$  pair of hadronic transverse size off the proton. The harder of the two quarks is essentially a spectator to diffractive scattering. The scattering of the softer quark off the proton is non-perturbative and cannot be described by exchange of a finite number of gluons. Hence there is an unsuppressed probability that the softer quark leaves the proton intact. This explains the leading twist nature of hard diffraction. The details of the scattering of the softer quark off the proton are encoded in the diffractive quark distribution. In a similar

way, the  $q\bar{q}g$  configuration in the virtual photon, in which the  $q\bar{q}$  pair carries almost all momentum, gives rise to the diffractive gluon distribution.

Because the short-distance cross section  $\hat{\sigma}^{\gamma^*i}$  of hard diffractive DIS is identical to inclusive DIS, the evolution of the diffractive parton distributions is identical to those of ordinary parton distributions. It follows that the characteristics of diffraction are entirely contained in the input distributions at a given scale. It is therefore interesting to model these distributions. The original idea of a partonic content of the pomeron [86] can be interpreted as an ansatz in which the diffractive parton distribution factorizes into a pomeron flux factor, which determines the  $\xi$  dependence, and a parton distribution in the pomeron which depends only on  $\beta = x/\xi$ . The precise data from HERA do not support this simple ansatz any more, although the problem can be fixed by adding more Regge poles. More recent approaches model the proton field off which the Fock states of the virtual photons scatter. The semi-classical approach [93], which preceded the factorization theorem, can be formulated in such a way that it models the diffractive parton distributions [94]. It can be justified for a large nucleus [95]. Applied to the proton it gives a reasonable description of both diffractive and inclusive DIS [96]. (See [97] for earlier work that contains some elements of the semi-classical approach.) Another approach is based on two gluon exchange [92, 98]. In this case one either has to deal with an infrared divergence, or couple the gluons to a small size toy nucleon as in [99]. Remarkably, these three approaches give similar results on the  $\beta$ -dependence of diffractive parton distributions and agree on the fact that the gluon distribution is enhanced by a large colour factor. This leads to positive scaling violations already at relatively large  $\beta$ , different from inclusive DIS, but in agreement with data.

It is encouraging that simple models reproduce the gross features of the data. Given the differences of the models as far as the proton is concerned, it seems that hard diffraction probes the wave-function of a virtual photon rather than the structure of the proton!

Hard diffraction in hadron-hadron collisions is much harder to describe and more varied, as there can be rapidity gaps between jets, between a jet and a hadron remnant etc.. Factorization does not seem to hold in this case [100], neither is it expected to [89], since, for example, an elastically scattered hadron must traverse the remnant of the other hadron, which can cause its break-up. I would like to note, however, a recent suggestion [101] to describe rapidity gap-like events (between jets) in terms of small energy flow in the gap rather than the absence of particles. Although this does not correspond exactly to the notion of hard diffractive scattering, such a definition is more appropriate for a partonic interpretation.

## 4.2 Skewed processes

Factorization has also been shown for deeply virtual Compton scattering  $\gamma^*p \rightarrow \gamma p$  [102] and diffractive vector meson production [103] (after earlier work in [104])  $\gamma(Q)p \rightarrow Vp$ , where  $V$  can be an onium and  $Q$  arbitrary or  $V$  can be a longitudinally polarized light vector meson, in which case  $Q^2$  must be large. Note that two-gluon exchange is applicable to diffractive vector meson production, but not to diffractive DIS, because convolution with the virtual photon wave-function relevant to longitudinal vector meson production suppresses the asymmetric  $q\bar{q}$  fluctuations, which have large transverse size. As a consequence, only the small size  $q\bar{q}$  component contributes at leading power.

Deeply virtual Compton scattering and diffractive vector meson production require a generalized parton distribution on the amplitude level, since the proton is scattered with non-zero momentum transfer, owing to the difference in invariant mass of the initial and final vector particle. These objects, defined as

$$p^+ \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle \quad (16)$$

for quarks, are referred to as skewed (off-diagonal, non-forward, ...) parton distributions, and describe a parton  $i$  (a quark above) extracted from the proton with momentum fraction  $x$  and returned with momentum fraction  $x'$ . For  $p' = p$ , the skewed parton distribution reduces to the conventional one. The first moment, however, is related to a proton form factor. These hybrid properties are also reflected in the evolution properties. For  $x' > 0$  the evolution resembles DGLAP evolution. For  $x' < 0$ , the skewed parton distribution describes emission of a  $q\bar{q}$  pair and the evolution resembles ERBL [105] evolution of light cone distribution amplitudes. The evolution properties and the form of skewed parton densities have been actively studied. An interesting observation is that the skewed parton density is determined by the conventional one for small  $x$  and  $x' - x$  [106].

Is there experimental evidence for skewedness? ZEUS [107] reports first evidence for deeply virtual Compton scattering, but the data are not yet good enough to allow detailed tests. Skewedness effects in diffractive vector meson production are largest if the invariant mass difference between the incoming photon and outgoing vector meson is large. This suggests to look at  $\Upsilon$  photoproduction [108] or vector meson production at large  $Q^2$  [109]. Incorporation of skewedness improves the theoretical prediction in comparison with data, but other theoretical uncertainties remain large and preclude an unambiguous statement. The power behaviour of longitudinal to transverse  $\rho$  meson production appears to disagree with the naive estimate  $\sigma_L/\sigma_T \sim Q^2$ , but the (formally logarithmic) scale-dependence of the gluon distribution may play an important role [109].



### 4.3 Exclusive $B$ decays

There exists a standard framework to discuss exclusive processes at large momentum transfer in terms of light cone distribution amplitudes [105]. As we are entering the era of exclusive  $B$  decays, it is only appropriate to consider them as bona fide hard reactions. After all they involve momentum transfers  $q^2 \sim m_b^2 \sim 25 \text{ GeV}^2$ , and there will be millions of  $B$ 's! A general two-body decay amplitude can be written as

$$\mathcal{A}(B \rightarrow M_1 M_2) = A_1 e^{i\delta_1} e^{i\delta_{W1}} + A_2 e^{i\delta_2} e^{i\delta_{W2}}, \quad (17)$$

where  $A_{1,2}$  denote the magnitudes of the amplitudes,  $\delta_{1,2}$  strong interaction phases and  $\delta_{W1,2}$  weak phases. The weak phases are CP-violating and of primary interest. Yet to determine them, the strong phases and amplitudes must be known, unless we are fortunate enough that only a single term contributes on the right hand side of (17), or we have enough experimental information to reduce strong interaction input.

The standard formalism does not immediately apply to  $B$  decays, because the  $B$  meson contains a soft spectator quark. The spectator quark may go to a final state meson without participating in a hard scattering. Hence the process cannot be described in terms of light cone distribution amplitudes alone. A more general factorization for decays into a heavy ( $D$ ) and a light meson has been proposed in [110], based on the idea that the light meson is initially ejected as a compact object from the weak decay vertex, although no quantitative conclusions have been drawn, with the exception of [111]. Recently, a systematic investigation of the heavy quark limit has been undertaken [112]. The conclusion is that all soft and collinear configurations can be absorbed either into light cone distribution amplitudes or a form factor for the transition  $B \rightarrow M_1$ , where  $M_1$  is the meson that picks up the light spectator quark. In particular, the corrections conventionally termed “non-factorizable” are dominated by hard gluon exchange and hence computable. The proposed factorization theorem, applicable to heavy-light final states ( $D\pi$ , etc.) and light-light final states ( $\pi\pi$ ,  $\pi K$ , etc.), reads

$$\begin{aligned} \mathcal{A}(B \rightarrow M_1 M_2) &= F_{B \rightarrow M_1}(0) \int_0^1 dx T^I(x) \Phi_{M_2}(x) \\ &+ \int_0^1 d\xi dx dy T^{II}(\xi, x, y) \Phi_B(\xi) \Phi_{M_1}(y) \Phi_{M_2}(x), \end{aligned} \quad (18)$$

with corrections that are suppressed as  $\Lambda_{\text{QCD}}/m_b$ . The second term is present only for light-light final states and has the form of a standard BL-type term. It accounts for hard gluon interactions with the spectator quark [113].

The implications of (18) taken at face value are far-reaching. Since non-perturbative form factors and light cone distribution amplitudes can either be measured or determined in principle with lattice QCD, the strong phases  $\delta_{1,2}$  and amplitudes

$A_{1,2}$  are completely predicted. CKM parameters can then be directly extracted from measurements of branching fractions and CP asymmetries.

Some work remains to be done to demonstrate that (18) gives accurate predictions at the  $b$  quark scale. A factorization proof to all orders has yet to be given, which may imply that an integration over intrinsic transverse momentum in the  $B$  meson has to be added to the second term of (18). Power corrections in  $\Lambda_{\text{QCD}}/m_b$  can turn out uncomfortably large, if enhanced by small current quark masses. It is also worth noting that the light cone properties of  $B$  mesons have remained largely unexplored. In any event, the new approach improves over naive factorization, which has been the most commonly used theoretical tool. Because of its potential for  $B$  factories, applications need to be carefully examined.

## Conclusion

QCD is a lively field of incredible variety. It is also often technical. Comparing today's QCD overviews with the discussion of big ideas 25 years ago, this variety may even appear intimidating. But this transformation in style reflects like no other indicator the progress in understanding how QCD works. The challenges provided by strong coupling have led to insights into how field theory works unparalleled by any other theory. Given the intrinsic beauty and simplicity of QCD, together with its role in the future high energy physics programme at the energy frontier, we can be sure of further progress in the field.

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## References

- [1] See, for example, the review R. Harlander and M. Steinhauser, Prog. Part. Nucl. Phys. **43**, 167 (1999).
- [2] S. G. Gorishny, A. L. Kataev and S. A. Larin, Phys. Lett. **B259**, 144 (1991); L. R. Surguladze and M. A. Samuel, Phys. Rev. Lett. **66**, 560 (1991); S. A. Larin and J. A. Vermaseren, Phys. Lett. **B259**, 345 (1991); S. A. Larin, T. van Ritbergen and J. A. Vermaseren, Phys. Lett. **B404**, 153 (1997).
- [3] T. van Ritbergen, J. A. Vermaseren and S. A. Larin, Phys. Lett. **B400**, 379 (1997).

- [4] K. G. Chetyrkin, Phys. Lett. **B404**, 161 (1997); J. A. Vermaseren, S. A. Larin and T. van Ritbergen, Phys. Lett. **B405**, 327 (1997).
- [5] F. V. Tkachov, Phys. Lett. **B100**, 65 (1981); K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. **B192**, 159 (1981).
- [6] A. A. Vladimirov, Theor. Mat. Fiz. **43**, 210 (1980); K. G. Chetyrkin and V. A. Smirnov, Phys. Lett. **B144**, 419 (1984).
- [7] D. J. Broadhurst, Z. Phys. **C54**, 599 (1992).
- [8] K. G. Chetyrkin, F. V. Tkachev and S. G. Gorishny, Phys. Lett. **B119**, 407 (1982); K. G. Chetyrkin, Theor. Math. Phys. **75**, 346 (1988); *ibid.* **76**, 809 (1988); S. G. Gorishny, Nucl. Phys. **B319**, 633 (1989); V. A. Smirnov, Commun. Math. Phys. **134**, 109 (1990).
- [9] V. A. Smirnov, Phys. Lett. **B394**, 205 (1997); A. Czarnecki and V. A. Smirnov, Phys. Lett. **B394**, 211 (1997).
- [10] M. Beneke and V. A. Smirnov, Nucl. Phys. **B522**, 321 (1998).
- [11] V. A. Smirnov, Phys. Lett. **B404**, 101 (1997); V. A. Smirnov and E. R. Rakhmetov, Theor. Math. Phys. **120**, 870 (1999).
- [12] K. G. Chetyrkin and M. Steinhauser, Phys. Rev. Lett. **83**, 4001 (1999).
- [13] S. Laporta and E. Remiddi, Phys. Lett. **B379**, 283 (1996).
- [14] K. Melnikov and T. van Ritbergen, [hep-ph/9912391].
- [15] K. G. Chetyrkin, J. H. Kühn and M. Steinhauser, Phys. Lett. **B371**, 93 (1996); Nucl. Phys. **B505**, 40 (1997).
- [16] A. Czarnecki and K. Melnikov, Phys. Rev. Lett. **80**, 2531 (1998).
- [17] A. Czarnecki and K. Melnikov, Nucl. Phys. **B505**, 65 (1997); Phys. Rev. Lett. **78**, 3630 (1997); Phys. Rev. **D59**, 014036 (1999).
- [18] A. Czarnecki and K. Melnikov, Nucl. Phys. **B544**, 520 (1999); K. G. Chetyrkin, R. Harlander, T. Seidensticker and M. Steinhauser, Phys. Rev. **D60**, 114015 (1999).
- [19] T. van Ritbergen, Phys. Lett. **B454**, 353 (1999).

- [20] A. Signer and L. Dixon, Phys. Rev. Lett. **78**, 811 (1997); L. Dixon and A. Signer, Phys. Rev. **D56**, 4031 (1997); Z. Nagy and Z. Trocsanyi, Phys. Rev. Lett. **79**, 3604 (1997); J. M. Campbell, M. A. Cullen and E. W. Glover, Eur. Phys. J. **C9**, 245 (1999); S. Weinzierl and D. A. Kosower, Phys. Rev. **D60**, 054028 (1999).
- [21] W. Bernreuther, A. Brandenburg and P. Uwer, Phys. Rev. Lett. **79**, 189 (1997); A. Brandenburg and P. Uwer, Nucl. Phys. **B515**, 279 (1998); G. Rodrigo, A. Santamaria and M. Bilenky, Phys. Rev. Lett. **79**, 193 (1997); P. Nason and C. Oleari, Phys. Lett. **B407**, 57 (1997).
- [22] P. Abreu *et al.* [DELPHI Collaboration], Phys. Lett. **B418**, 430 (1998); A. Brandenburg, P. N. Burrows, D. Muller, N. Oishi and P. Uwer, Phys. Lett. **B468**, 168 (1999); DELPHI collaboration, contribution to EPS99, Tampere, Finland, 15-21 July 1999.
- [23] W. B. Kilgore and W. T. Giele, Phys. Rev. **D55**, 7183 (1997); Z. Trocsanyi, Phys. Rev. Lett. **77**, 2182 (1996).
- [24] J. M. Campbell and E. W. N. Glover, Nucl. Phys. **B527**, 264 (1998); S. Catani and M. Grazzini, Phys. Lett. **B446**, 143 (1999); [hep-ph/9908523]; V. Del Duca, A. Frizzo and F. Maltoni, [hep-ph/9909464].
- [25] Z. Bern, V. Del Duca and C. R. Schmidt, Phys. Lett. **B445**, 168 (1998); Z. Bern, V. Del Duca, W. B. Kilgore and C. R. Schmidt, Phys. Rev. **D60**, 116001 (1999); D. A. Kosower and P. Uwer, [hep-ph/9903515].
- [26] V. A. Smirnov, Phys. Lett. **B460**, 397 (1999).
- [27] V. A. Smirnov and O. L. Veretin, [hep-ph/9907385].
- [28] J. B. Tausk, Phys. Lett. **B469**, 225 (1999).
- [29] See, for example, the review Z. Bern, L. Dixon and D. A. Kosower, Ann. Rev. Nucl. Part. Sci. **46**, 109 (1996).
- [30] Z. Bern, J. S. Rozowsky and B. Yan, Phys. Lett. **B401**, 273 (1997).
- [31] Z. Bern, L. Dixon and D. A. Kosower, [hep-ph/0001001].
- [32] S. Catani, Phys. Lett. **B427**, 161 (1998).
- [33] S. A. Larin, T. van Ritbergen and J. A. Vermaseren, Nucl. Phys. **B427**, 41 (1994); S. A. Larin, P. Nogueira, T. van Ritbergen and J. A. Vermaseren, Nucl. Phys. **B492**, 338 (1997).
- [34] W. L. van Neerven and A. Vogt, [hep-ph/9907472].

- [35] G. 't Hooft, in *The whys of subnuclear physics*, Erice Lectures 1977, A. Zichichi (ed.); A. H. Mueller, Nucl. Phys. **B250**, 327 (1985).
- [36] See, for example, the review M. Beneke, Phys. Rept. **317**, 1 (1999).
- [37] A. V. Manohar and M. B. Wise, Phys. Lett. **B344**, 407 (1995); B. R. Webber, Phys. Lett. **B339**, 148 (1994); G. P. Korchemsky and G. Sterman, Nucl. Phys. **B437**, 415 (1995).
- [38] Y. L. Dokshitzer and B. R. Webber, Phys. Lett. **B352**, 451 (1995); R. Akhoury and V. I. Zakharov, Phys. Lett. **B357**, 646 (1995).
- [39] P. Nason and M. H. Seymour, Nucl. Phys. **B454**, 291 (1995); M. Beneke and V. M. Braun, Nucl. Phys. **B454**, 253 (1995).
- [40] Y. L. Dokshitzer and B. R. Webber, Phys. Lett. **B404**, 321 (1997).
- [41] Y. L. Dokshitzer, G. Marchesini and G. P. Salam, Eur. Phys. J. direct **C3**, 1 (1999).
- [42] Y. L. Dokshitzer, A. Lucenti, G. Marchesini and G. P. Salam, Nucl. Phys. **B511**, 396 (1998); JHEP **9805**, 003 (1998).
- [43] M. Beneke, V. M. Braun and L. Magnea, Nucl. Phys. **B497**, 297 (1997).
- [44] M. Dasgupta, L. Magnea and G. Smye, JHEP **9911**, 025 (1999).
- [45] P. A. Movilla Fernandez, O. Biebel and S. Bethke, contribution to EPS99, Tampere, Finland, 15-21 July 1999 [hep-ex/9906033]; L3 Collaboration, contribution to EPS99, Tampere, Finland, 15-21 July 1999; DELPHI Collaboration, contribution to EPS99, Tampere, Finland, 15-21 July 1999.
- [46] K. Rabbertz [H1 Collaboration], contribution to DIS 99, Zeuthen, Germany, 19-23 April 1999 [hep-ex/9906002].
- [47] G. P. Korchemsky, G. Oderda and G. Sterman, contribution to DIS 97, Chicago, IL, 14-18 April 1997 [hep-ph/9708346]; G. P. Korchemsky and G. Sterman, Nucl. Phys. **B555**, 335 (1999).
- [48] N. A. Sveshnikov and F. V. Tkachev, Phys. Lett. **B382**, 403 (1996).
- [49] F. Di Renzo, E. Onofri and G. Marchesini, Nucl. Phys. **B457**, 202 (1995); G. Burgo, F. Di Renzo, M. Pepe and L. Scorzato, contribution to LATTICE 99, Pisa, Italy, 29 June - 3 July 1999 [hep-lat/9909169].

- [50] G. Burgio, F. Di Renzo, G. Marchesini and E. Onofri, Phys. Lett. **B422**, 219 (1998).
- [51] M. Beneke, V. M. Braun and N. Kivel, Phys. Lett. **B443**, 308 (1998).
- [52] R. Akhoury and V. I. Zakharov, Phys. Lett. **B438**, 165 (1998).
- [53] G. Sterman, Nucl. Phys. **B281**, 310 (1987); S. Catani and L. Trentadue, Nucl. Phys. **B327**, 323 (1989).
- [54] S. Catani, L. Trentadue, G. Turnock and B. R. Webber, Nucl. Phys. **B407**, 3 (1993).
- [55] N. Kidonakis and G. Sterman, Phys. Lett. **B387**, 867 (1996); Nucl. Phys. **B505**, 321 (1997).
- [56] R. Bonciani, S. Catani, M. L. Mangano and P. Nason, Nucl. Phys. **B529**, 424 (1998).
- [57] N. Kidonakis and R. Vogt, Phys. Rev. **D59**, 074014 (1999); E. Laenen and S. Moch, Phys. Rev. **D59**, 034027 (1999).
- [58] S. Catani, M. L. Mangano, P. Nason, C. Oleari and W. Vogelsang, JHEP **9903**, 025 (1999); N. Kidonakis and J. F. Owens, [hep-ph/9912388].
- [59] N. Kidonakis, G. Oderda and G. Sterman, Nucl. Phys. **B525**, 299 (1998); *ibid.* **B531**, 365 (1998).
- [60] L. Apanasevich *et al.* [Fermilab E706 Collaboration], Phys. Rev. Lett. **81**, 2642 (1998).
- [61] B. A. Thacker and G. P. Lepage, Phys. Rev. **D43**, 196 (1991); G. P. Lepage, L. Magnea, C. Nakhleh, U. Magnea and K. Hornbostel, Phys. Rev. **D46**, 4052 (1992).
- [62] A. Pineda and J. Soto, Nucl. Phys. Proc. Suppl. **64**, 428 (1998) [hep-ph/9707481].
- [63] A. H. Hoang and T. Teubner, Phys. Rev. **D58**, 114023 (1998); K. Melnikov and A. Yelkhovsky, Nucl. Phys. **B528**, 59 (1998); O. Yakovlev, Phys. Lett. **B457**, 170 (1999).
- [64] M. Beneke, A. Signer and V. A. Smirnov, Phys. Lett. **B454**, 137 (1999).
- [65] A. H. Hoang and T. Teubner, Phys. Rev. **D60**, 114027 (1999);
- [66] T. Nagano, A. Ota and Y. Sumino, Phys. Rev. **D60**, 114014 (1999); A. A. Penin and A. A. Pivovarov, [hep-ph/9904278].

- [67] A. A. Penin and A. A. Pivovarov, Phys. Lett. **B435**, 413 (1998); A. H. Hoang, Phys. Rev. **D59**, 014039 (1999).
- [68] K. Melnikov and A. Yelkhovsky, Phys. Rev. **D59**, 114009 (1999); A. H. Hoang, Phys. Rev. **D61**, 034005 (2000); M. Beneke and A. Signer, [hep-ph/9906475].
- [69] N. Brambilla, A. Pineda, J. Soto and A. Vairo, [hep-ph/9910238]; B. A. Kniehl and A. A. Penin, [hep-ph/9907489]; [hep-ph/9911414]; M. E. Luke, A. V. Manohar and I. Z. Rothstein, [hep-ph/9910209]; A. V. Manohar and I. W. Stewart, [hep-ph/9912226].
- [70] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977); I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
- [71] V. S. Fadin and L. N. Lipatov, Phys. Lett. **B429**, 127 (1998); M. Ciafaloni and G. Camici, Phys. Lett. **B430**, 349 (1998).
- [72] M. Acciarri *et al.* [L3 Collaboration], Phys. Lett. **B453**, 333 (1999); OPAL collaboration, Physics Note 391, contribution to this symposium.
- [73] C. Adloff *et al.* [H1 Collaboration], Phys. Lett. **B462**, 440 (1999).
- [74] D. A. Ross, Phys. Lett. **B431**, 161 (1998).
- [75] G. P. Salam, JHEP **9807**, 019 (1998).
- [76] B. Andersson, G. Gustafson and J. Samuelsson, Nucl. Phys. **B467**, 443 (1996); C. R. Schmidt, Phys. Rev. **D60**, 074003 (1999).
- [77] J. R. Forshaw, D. A. Ross and A. S. Vera, Phys. Lett. **B455**, 273 (1999).
- [78] M. Ciafaloni and D. Colferai, Phys. Lett. **B452**, 372 (1999).
- [79] M. Ciafaloni, D. Colferai and G. P. Salam, Phys. Rev. **D60**, 114036 (1999).
- [80] G. Altarelli, R. D. Ball and S. Forte, [hep-ph/9911273].
- [81] Y. V. Kovchegov and A. H. Mueller, Phys. Lett. **B439**, 428 (1998); E. Levin, [hep-ph/9806228].
- [82] R. S. Thorne, Phys. Rev. **D60**, 054031 (1999); [hep-ph/9912284].
- [83] M. Ciafaloni, D. Colferai and G. P. Salam, JHEP **9910**, 017 (1999).
- [84] For a comprehensive discussion of diffraction see the talk by H. Abramowicz at this symposium [hep-ph/0001054].

- [85] R. Bonino *et al.* [UA8 Collaboration], Phys. Lett. **B211**, 239 (1988).
- [86] G. Ingelman and P. E. Schlein, Phys. Lett. **B152**, 256 (1985).
- [87] M. Derrick *et al.* [ZEUS Collaboration], Phys. Lett. **B315**, 481 (1993); T. Ahmed *et al.* [H1 Collaboration], Nucl. Phys. **B429**, 477 (1994). C. Adloff *et al.* [H1 Collaboration], Z. Phys. **C76**, 613 (1997); J. Breitweg *et al.* [ZEUS Collaboration], Eur. Phys. J. **C6**, 43 (1999).
- [88] L. Trentadue and G. Veneziano, Phys. Lett. **B323**, 201 (1994); A. Berera and D. E. Soper, Phys. Rev. **D50**, 4328 (1994).
- [89] A. Berera and D. E. Soper, Phys. Rev. **D53**, 6162 (1996).
- [90] M. Grazzini, L. Trentadue and G. Veneziano, Nucl. Phys. **B519**, 394 (1998); J. C. Collins, Phys. Rev. **D57**, 3051 (1998).
- [91] J. D. Bjorken and J. B. Kogut, Phys. Rev. **D8**, 1341 (1973).
- [92] N. N. Nikolaev and B. G. Zakharov, Z. Phys. **C49**, 607 (1991);
- [93] W. Buchmüller and A. Hebecker, Nucl. Phys. **B476**, 203 (1996); W. Buchmüller, M. F. McDermott and A. Hebecker, Nucl. Phys. **B487**, 283 (1997); *ibid.* **500**, 621 (1997) (E).
- [94] A. Hebecker, Nucl. Phys. **B505**, 349 (1997).
- [95] A. Hebecker and H. Weigert, Phys. Lett. **B432**, 215 (1998).
- [96] W. Buchmüller, T. Gehrmann and A. Hebecker, Nucl. Phys. **B537**, 477 (1999).
- [97] O. Nachtmann, Annals Phys. **209**, 436 (1991); A. Krämer and H. G. Dosch, Phys. Lett. **B252**, 669 (1990).
- [98] N. N. Nikolaev and B. G. Zakharov, Z. Phys. **C64**, 631 (1994); J. Bartels and M. Wüsthoff, J. Phys. **G22**, 929 (1996); M. Wüsthoff and A. D. Martin, J. Phys. **G25**, R309 (1999).
- [99] F. Hautmann, Z. Kunszt and D. E. Soper, Phys. Rev. Lett. **81**, 3333 (1998); [hep-ph/9906284].
- [100] L. Alvero, J. C. Collins, J. Terron and J. J. Whitmore, Phys. Rev. **D59**, 074022 (1999).
- [101] G. Oderda and G. Sterman, Phys. Rev. Lett. **81**, 3591 (1998).



- [102] D. Müller, D. Robaschik, B. Geyer, F. M. Dittes and J. Horejsi, Fortschr. Phys. **42**, 101 (1994); X. Ji, Phys. Rev. Lett. **78**, 610 (1997); Phys. Rev. **D55**, 7114 (1997); A. V. Radyushkin, Phys. Lett. **B380**, 417 (1996); J. C. Collins and A. Freund, Phys. Rev. **D59**, 074009 (1999).
- [103] J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. **D56**, 2982 (1997).
- [104] M. G. Ryskin, Z. Phys. **C57**, 89 (1993); S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. **D50**, 3134 (1994).
- [105] A. V. Efremov and A. V. Radyushkin, Phys. Lett. **B94**, 245 (1980); G. P. Lepage and S. J. Brodsky, Phys. Rev. **D22**, 2157 (1980).
- [106] A. G. Shuvaev, K. J. Golec-Biernat, A. D. Martin and M. G. Ryskin, Phys. Rev. **D60**, 014015 (1999).
- [107] ZEUS Collaboration, contribution to EPS99, Tampere, Finland, 15-21 July 1999.
- [108] L. L. Frankfurt, M. F. McDermott and M. Strikman, JHEP **9902**, 002 (1999).
- [109] A. D. Martin, M. G. Ryskin and T. Teubner, [hep-ph/9912551].
- [110] J. D. Bjorken, Nucl. Phys. Proc. Suppl. **11**, 325 (1989); M. J. Dugan and B. Grinstein, Phys. Lett. **B255**, 583 (1991).
- [111] H. D. Politzer and M. B. Wise, Phys. Lett. **B257**, 399 (1991).
- [112] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999).
- [113] A. Szczepaniak, E. M. Henley and S. J. Brodsky, Phys. Lett. **B243**, 287 (1990).

## Discussion

**Günter Grindhammer (MPI, Munich):** Considering deep inelastic scattering at low  $x$ , what happens with the difference between the BFKL and DGLAP approaches in the case of heavy quark production? In this case one has two hard scales,  $Q^2$  and the quark mass.

The answer depends on where the heavy quarks are. If the heavy quarks couple to the virtual photon (“the top of the ladder”), there is no difference to inclusive DIS

at small  $x$ . If the heavy quark pair is coupled to the bottom of the ladder, both ends are perturbative, as is the case for forward jet production at  $p_T \gg \Lambda_{\text{QCD}}$ . In this case there is no reason to believe that DGLAP evolution should be relevant.